

# Mesoscopic continuous and discrete channels for quantum information transfer

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We study the possibility of realizing perfect quantum state transfer in mesoscopic devices. We discuss the case of the Fano-Anderson model extended to two impurities. For a channel with an infinite number of degrees of freedom, we obtain coherent behavior in the case of strong coupling or in weak coupling off-resonance. For a finite number of degrees of freedom, coherent behavior is associated to weak coupling and resonance conditions.

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Quantum information and quantum communication require the ability of manipulating and transfer qubits in the space [1]. Quantum state transfer (QST) can be realized by teleportation [2], using flying qubits [4], or through quantum channels. When the information has to be processed in devices smaller than typical optical wavelengths for flying qubits, quantum channels are preferable. They can be based on solid state devices or on confined radiation fields. QST among optical cavities, as proposed by Cirac *et al.* some years ago [4], is possible due to the fact that each atom inside the cavity interacts only with a nearly monochromatic photon of the radiation field, and that photon can be transmitted unchanged to a distant site, before interacting with another atom in a second cavity. Generally, in mesoscopic devices, an interaction localized in the space involves all the modes of the support and the state reconstruction is affected by interference.

The use of local excitations in quantum chains, first suggested by Bose [3], is indeed far from being optimal, due to quantum diffusion [5, 6]. Different physical realizations of quantum channels have been suggested: ferromagnetic spin chains [3, 7], Josephson arrays [8], nanoelectromechanical oscillators [9]. Quantum diffusion appears in each of these models. Ideally, this drawback can be overcome by using parallel chains and conditional gates [10] or through the adoption of engineered couplings between the nodes of the network [11]. Practically, these solutions are rather complicated to be realized. Recently, two schemes have been proposed in which the quantum chain act as a quantum bus, and both the encoding site and the decoding site are external and coupled locally with two different points of the chain [12, 13]. In Ref.[12] the coherent behavior of the channel is associated with a simple model which takes into account only the center of mass mode of the chain. Numerical and analytical considerations show that the transmission in a finite-size chain can be nearly perfect and the speed of propagation is independent from the distance between sender and receiver, but only depends on the total length of the chain.

The latter is one of possible models, where two (or

more) quantum systems interact via an intermediate channel characterized by many degrees of freedom.

In this Letter we study the general conditions under which such systems exhibit quantum oscillations, and therefore are suitable for QST, in the limit of discrete or continuous channel.

For the sake of concreteness, we consider the Fano-Anderson model [14, 15] extended to two impurities:

$$H = \sum_k \epsilon_k c_k^\dagger c_k + \Omega \left( c_A^\dagger c_A + c_B^\dagger c_B \right) - \frac{g}{\sqrt{N}} \sum_k \left[ c_k^\dagger (c_A + e^{ikL} c_B) + H.c. \right]. \quad (1)$$

We have two quantum systems (*A* and *B*) with creation and annihilation operators  $c_A^\dagger, c_A, c_B^\dagger, c_B$ , a chain with *N* modes, described by  $c_k^\dagger$  ( $c_k$ ) which creates (annihilates) an excitation in the mode *k*, and interaction with the modes and *A* and *B* which amounts to tunneling processes in the case when both *A* and *B* are associated with a solid state tight binding model, or to a transfer of energy when *A* and *B* are atomic systems interacting with a radiation field. The coupling constant *g* measures the strength of the interaction and the phase factor  $\exp(ikL)$  takes into account the distance *L* between *A* and *B*. In the case of a continuous spectrum, sums must be thought as integrals. Due to the quadratic nature of the Hamiltonian, the evolution equation of each operator is independent from the corresponding quantum statistics. Then, the model works for fermions as well as for bosons. All the characteristics of the system are synthesized by the energy dispersion  $\epsilon_k$ .

In the case of continuum of states, possible candidates as mesoscopic channels are conductors in the tight binding limit or one-dimensional wires with magnetic edge states [16], where there are experimental proofs of coherent hopping with quantum dots [17, 18]. As far as discrete sets of states are considered, the model is suitable to be implemented by arrays of quantum dots, or by nanoelectromechanical oscillators, or by a radiation confined in a finite-size cavity. An experimental evidence of coherent oscillations in an all solid state realization of a

Jaynes-Cummings-like scheme has been recently reported [19].

We will show that coherent oscillations between  $A$  and  $B$  can be achieved using both continuous and discrete channels. In particular, discrete channels are suitable for our purposes when  $A$  and  $B$  are weakly coupled with the chain and  $\Omega$  is resonant with one of its eigenvalues  $\epsilon_k$ . In this situation, only the resonant modes play a significant role and the effective Hamiltonian is that of a few-body problem. The same behavior can be attained

with continuous channels in the case of strong coupling, or, in the weak coupling limit, whenever  $\Omega$  lies outside the energy band.

Let us start with an initial state where an excitation is present in the impurity  $A$  and both the second impurity and the channel are in their respective vacuum states:  $|\psi_{in}\rangle = c_A^\dagger |0\rangle$ . Writing the Heisenberg equations and replacing the operators with their Laplace transform  $\tilde{c}^\dagger(\omega) = \int_0^\infty e^{i\omega t} c^\dagger(t) dt$ , assuming  $\hbar = 1$ , we find

$$\tilde{c}_A^\dagger(\omega) = \frac{i}{D(\omega)} \left\{ [\omega - \Omega - \Lambda_0(\omega)] \left( c_A^\dagger - \frac{g}{\sqrt{N}} \sum_k \frac{1}{\omega - \epsilon_k} c_k^\dagger \right) + \Lambda_L(\omega) \left( c_B^\dagger - \frac{g}{\sqrt{N}} \sum_k \frac{e^{ikL}}{\omega - \epsilon_k} c_k^\dagger \right) \right\}, \quad (2)$$

where

$$\Lambda_d(\omega) = \frac{g^2}{N} \sum_k \frac{e^{ikd}}{\omega - \epsilon_k}, \quad (3)$$

and  $D(\omega) = [\omega - \Omega - \Lambda_0(\omega)]^2 - \Lambda_L^2(\omega)$ . In Eq. (2), terms in  $c_k^\dagger$ , due to the presence of the excitation in the channel, introduce noise and limit the efficiency of the channel.

Studying the zeroes of the spectral function  $D(\omega)$ , we extract all information about the system. We can assume  $\epsilon_k = -w \cos ka$ , as usual when treating solids with many atoms and lattice constant  $a$ , and interpret  $A$  and  $B$  as impurity states. Here,  $2w$  is the bandwidth and  $k$  is defined in the first Brillouin zone limited by 0 and  $2\pi$ :  $k = 2\pi n/N$ , where  $n$  is any integer between 0 and  $N-1$ . Without loss of generality, we shall assume throughout the paper  $a=1$  and  $w=1$ . It can be shown that, in this case,

$$\Lambda_d(\omega_0) = \frac{g^2}{(\omega^2 - 1)^{1/2}} \frac{K_{-d}(\omega) + K_{N+d}(\omega)}{1 - K_N(\omega)}, \quad (4)$$

where  $K_d(\omega) = [-\omega + (\omega^2 - 1)^{1/2}]^d$ .

In order to evaluate its zeroes, the spectral function can be decomposed in two factors:  $D(\omega) = D_+(\omega)D_-(\omega)$ , where

$$D_\pm(\omega) = \omega - \Omega - \frac{g^2}{(\omega^2 - 1)^{1/2}} \times \frac{1 + K_N(\omega) \pm [K_{-L}(\omega) + K_{N+L}(\omega)]}{1 - K_N(\omega)}. \quad (5)$$

The analytic structure of Eq. (2) consists in  $2(N+1)$  real poles for every finite  $N$ , and, in the limit  $N \rightarrow \infty$ , only 4 real poles, related to the band extrema, remain, and poles inside the energy band are substituted by a cut.

These exact results are compared, in the following, with the direct numerical solution of the evolution equation.

We start from the weak coupling regime ( $g \ll 1$ ). The zeroes of Eq. (5) can be calculated by iterating the zero order solution  $\omega = \omega_0$  obtained in the limit  $g \rightarrow 0$ .

First, we assume  $\Omega$  outside the energy band:  $|\Omega| > 1$ . In this case the zero order solution is  $\omega_0 = \Omega$  and, by iteration,

$$\omega_{1,2} = \Omega + \Lambda_0(\Omega) \pm \Lambda_L(\Omega). \quad (6)$$

All roots are real and oscillations are expected. Residues associated to poles  $\omega_1$  and  $\omega_2$  in Eq.(2) are obtained neglecting terms in powers of order  $g^2$ . In such limit we find that all the spectral weight is concentrated on the impurities' modes. Then, we obtain a coherent oscillation between the two impurities:

$$c_A^\dagger(t) = e^{-\frac{i\omega_+ t}{2}} \left( \cos \frac{\omega_- t}{2} c_A^\dagger - i \sin \frac{\omega_- t}{2} c_B^\dagger \right), \quad (7)$$

where  $\omega_+ = 2[\Omega + \Lambda_0(\Omega)]$  and  $\omega_- = 2\Lambda_L(\Omega)$ . In the limit of infinite number of modes,  $\omega_+ = 2\Omega + 2g^2/(\Omega^2 - 1)^{1/2}$  and  $\omega_- = 2g^2[\Omega - \sqrt{\Omega^2 - 1}]^L/\sqrt{\Omega^2 - 1}$ . These solutions illustrate that the open system  $A + B$  experiences a Rabi oscillation, and actually behaves as a closed one. Then, the system is suitable for QST or to create entanglement. In the case discussed above the dependence on the size-system is not crucial and the continuous limit is achieved even for not very large values of  $N$ . In Fig. 1 we report the probabilities of the excitation to be localized either on the first impurity or on the second one.

The discussion becomes more interesting when  $|\Omega| < 1$ . In this case it can be useful to introduce an auxiliary complex variable  $\gamma$  defined by  $\omega = -\cos \gamma$ , with the con-

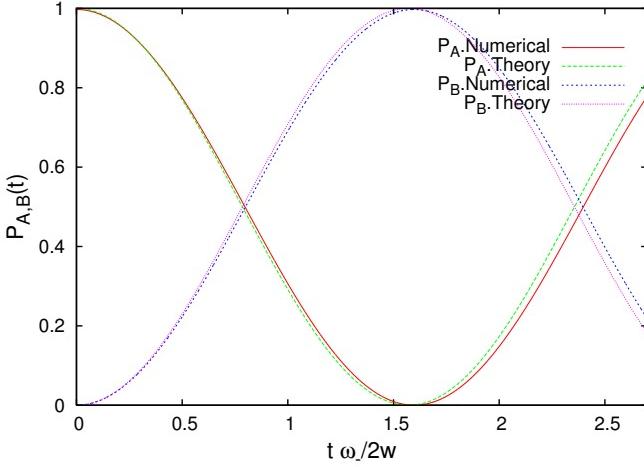


FIG. 1: Time evolution of the occupation probabilities of  $A$  and  $B$  ( $P_A$  and  $P_B$ ) in weak coupling and off-resonance. The coupling strength is  $g = 0.05$ , the impurities' energy is  $\Omega = 1.5$ , the number of the channel's elements is  $N = 30$ , and the distance between  $A$  and  $B$  is  $L = 6$ . Here are reported both the numerical (exact) and theoretical curves.

straint that  $0 \leq \text{Re}\{\gamma\} \leq \pi$ . So

$$D_{\pm}(\gamma) = \cos \gamma - \cos \Gamma + g^2 \frac{1}{\sin \gamma \sin \gamma N/2} [\cos \gamma N/2 \pm \cos \gamma(L + N/2)], \quad (8)$$

having defined  $\Omega = -\cos \Gamma$ . At the resonance,  $\Omega$  coincides with one of the unperturbed poles.

Since in the weak coupling limit the original energy levels are slightly modified, it is reasonable to assume that the resonant ones give the main contribution to the evolution and an expansion around them can be done. We write  $\gamma = \Gamma + \delta$ , with  $\delta$  expected to vanish in the limit of  $g = 0$ . Then

$$\delta \simeq \frac{g^2}{\sin^2 \Gamma} \left[ \cot \frac{\delta N}{2} (1 \pm \cos \Gamma L) \mp \sin \Gamma L \right]. \quad (9)$$

Two different regimes appear for  $\delta N \gg 1$  or  $\delta N \ll 1$ . In the first case the system is well approximated by its continuum limit, obtained replacing  $\cot \delta N/2$  with  $-i \text{ sign}\{Im\{\delta\}\}$ . It is easy to show that Eq. (9) does not provide polar solutions, but only singularities deriving from the cut. Under these conditions, the excitation diffuses in the channel and the QST efficiency is lost.

On the other hand, when  $\delta N \ll 1$  the cotangent in Eq.(9) is expanded into  $2/(\delta N)$  and  $\sin \Gamma L$  is negligible. The solutions are then  $\delta_1^{\pm} = \pm g \sqrt{2(1 - \cos \Gamma L)/N \sin^2 \Gamma}$  and  $\delta_2^{\pm} = \pm g \sqrt{2(1 + \cos \Gamma L)/N \sin^2 \Gamma}$ . The time evolution of  $c_A^\dagger$  looks very simple when  $\Omega = 0$  and  $L$  is even:

in such a case

$$c_A^\dagger(t) = \cos^2 \frac{gt}{\sqrt{N}} c_A^\dagger + (-1)^{1+L/2} \sin^2 \frac{gt}{\sqrt{N}} c_B^\dagger + \frac{i}{2} \sin \frac{2gt}{\sqrt{N}} (c_{\bar{k}}^\dagger + c_{-\bar{k}}^\dagger), \quad (10)$$

where  $\pm \bar{k}$  are the modes in resonance with  $\Omega = 0$ . This formula shows that, despite the non vanishing probability of finding the excitation in the channel, perfect QST is achieved. In Fig. 2 we report the time evolution of  $P_A$  and  $P_B$ , which represent the occupation probabilities of  $A$  and  $B$ . On the other hand, assuming  $L$  odd, the second impurity is never populated:

$$c_A^\dagger(t) = \cos gt \sqrt{\frac{2}{N}} c_A^\dagger + \frac{i}{\sqrt{2}} \sin gt \sqrt{\frac{2}{N}} (c_{\bar{k}}^\dagger + c_{-\bar{k}}^\dagger). \quad (11)$$

The result of Eq. (10) is somewhat similar to that obtained in Ref. [12], showing an efficiency of transfer independent (limiting ourselves to even values of  $L$ ) from the distance. The condition  $\delta N \ll 1$  (or  $g\sqrt{N} \ll 1$ ) can be interpreted as follows: the interaction splits the resonant pole in two levels with an energy separation of the order of  $g/\sqrt{N}$ , while the energy spacing between different modes is about  $1/N$ . If none of the other modes falls inside this energy interval, then the excitation interacts effectively only with the resonant modes and the coherent behavior appears. Vice versa, when  $g/\sqrt{N} \gg 1/N$ , the resonance is no more separated from the other modes and a continuum-like behavior is expected.

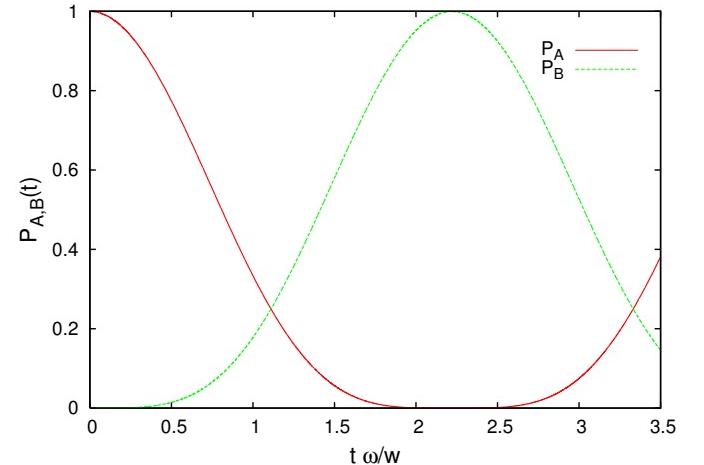


FIG. 2: Numerical simulation of the evolution of  $P_A(t)$  and  $P_B(t)$  in weak coupling and resonance with the following parameters:  $g = 0.01$ ,  $\Omega = 0$ ,  $N = 16$ , and  $L = 8$ . The time is normalized with respect to  $\omega = \sqrt{2g}/\sqrt{N}$ . The theoretical behavior, calculated in the text, coincides perfectly with the numerical one.

The other regime we want to explore is characterized by strong coupling ( $g \gg 1$ ). Now, we look for solutions of

the order of  $g$ , then  $\Omega$  does not matter, at least in a first approximation, and can be set to zero. In practice, due to the strong character of the interaction, the modified energies of impurities lie ever outside the band, and the channel works in its continuum limit even for not too large  $N$ . Considering  $\omega \gg 1$ , we obtain, by iterative procedure:

$$c_A^\dagger(t) = \cos gt \left[ \cos \frac{gt}{2(2g)^L} c_A^\dagger + (-1)^L i \sin \frac{gt}{2(2g)^L} c_B^\dagger \right] + \int dk f(k) c_k^\dagger \quad (12)$$

where  $f(k)$  is a function that satisfies the condition  $\int dk |f(k)|^2 = \sin^2 gt$ . In this case, we have high frequency oscillations between  $A$  and  $B$  and the channel modulated by a low frequency signal which enables QST. Note that the spectral weight is not entirely concentrated on the impurities, because at intermediate times the probability of finding the excitation in the channel is finite. Note also the different scaling with the distance  $L$  of the information carrying oscillation with respect to weak coupling. In Fig. 3 the probabilities of finding the excitation on  $A$  and  $B$  are depicted. The lower panel shows the high frequency oscillation. The discussion of this limit fails when infinitely extended discrete spectra are considered, as, for instance, in the case of finite-length cavities. In this situation, coherent behavior is expected to come out only from resonance conditions.

In conclusion, we have discussed by analytical and numerical calculations a number of possible configurations of a quantum bus allowing perfect state transfer or entanglement generation. The model we have considered is suitable for implementation of continuous or discrete quantum channels in different physical scenarios. We have illustrated in detail in which limits a coherent behavior emerges, showing that both weak coupling and strong coupling are suitable for our purposes. We hope that our conclusions will help to design an experimental setup.

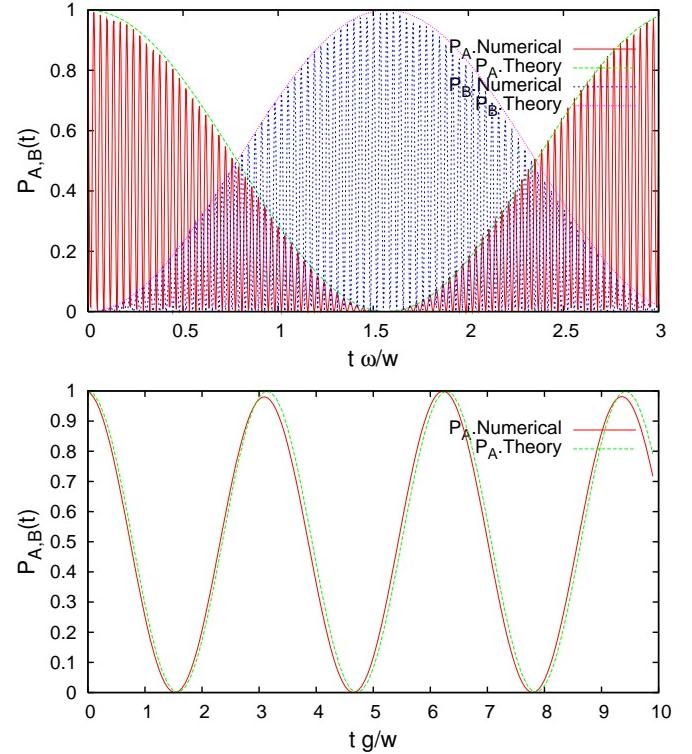


FIG. 3: Strong coupling limit:  $g = 10$ ,  $\Omega = 0$ ,  $L = 4$ ,  $N = 50$ . In the upper panel the low frequency oscillations are compared with the theory, the time unit is  $\omega = g/[2(2g)^L]$ . In the lower panel the same comparison is reported for the higher frequencies.

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